Stochastic Crack Propagation in Fastener Holes

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A simple crack-growth-rate-based stochastic model for fatigue crack propagation in fastener holes under spectrum loadings is investigated. With available fractographic data in the very small crack size region, i.e., 0.004 to 0.07 in., the model was demonstrated previously to be very good. Laboratory tests were conducted herein using wide fastener hole specimens to obtain fractographic data covering the small and large crack size regions in both laboratory air and a corrosive environment. The correlations between the stochastic model and the fractographic results in either the corrosive or noncorrosive environment are demonstrated to be very good. The model is shown to be valid for crack growth damage accumulation in fastener holes from the very small crack up to the critical crack size. Such a simple model is suitable for practical applications, such as in durability and damage tolerance analyses. Factors affecting the stochastic crack growth analysis and prediction are also investigated.

I. Introduction

FATIGUE cracking in fastener holes is one of the important failure modes in metallic airframes. In durability and damage tolerance analyses, the prediction of the fatigue crack growth damage accumulation plays an essential role. Experimental test results indicate that fatigue crack growth involves considerable statistical variability; such variability should be accounted for in the design of airframe structures.

Unfortunately, the statistical dispersion of the crack growth rate has been observed to vary with respect to many parameters, such as materials, amounts of load transfer, types of specimens, magnitude of constant amplitude loads, types of spectrum loadings, and ranges of crack size. For instance, the crack growth rate dispersion for specimens under constant amplitude loading differs from that under spectrum loading. The variability in crack growth damage accumulation for fastener hole specimens with natural cracks (starting from time to crack initiation) differs also from that of preflawed specimens. For practical analysis and design purposes, test results as close to the service environments as possible are highly desirable, and the statistical model should be established based on the correlation with test data thus obtained.

Fractographic data in the small crack size region for fastener hole specimens under fighter and bomber loading spectra were available in Ref. 1. A simple statistical model for the fatigue crack growth damage accumulation was proposed in Ref. 2, and the model was shown to have good correlation with the test results given in Ref. 1 in the small crack region. In the durability analysis for functional impairments, such as fuel leakage and ligament breakage, as well as the damage tolerant analysis, crack propagation in the large crack size region is of vital importance. Recently experimental tests have been conducted using 7475-T7351 aluminum fastener hole specimens subjected to both fighter and bomber spectra. The specimens are 3 in. wide so that fractographic data in the large crack size region are available. In addition, similar specimens have been tested in a corrosive environment to obtain fractographic data.[3,4]

The purpose of this paper is to show that the simple stochastic model for fatigue crack propagation proposed in Ref. 2 applies not only in the small crack size region (e.g., less than 0.1 in.) but also in the large crack size region for fastener hole specimens under spectrum loadings. The model is shown to be valid using crack propagation results acquired in both laboratory air and a corrosive environment. Good correlation is demonstrated between the stochastic model and the test results. Furthermore, factors affecting the accuracy of stochastic crack propagation analysis and prediction are also investigated herein. These factors include the data processing procedures for obtaining the crack growth rate data and the number of fractographic data points for each specimen. Recommendations are made to deal with these factors in the stochastic crack growth analysis.

II. Stochastic Model for Fatigue Crack Propagation

Various crack growth rate models have been proposed in the literature for predicting the crack growth damage accumulation (e.g., Refs. 5-7). These models have the general form

$$\frac{\mathrm{d}a(t)}{\mathrm{d}t}=L(\Delta K,a,K,R,S) \tag{1}$$

in which $a(t)$ = crack size, L = a non-negative function, ΔK = stress intensity factor range, K = stress intensity factor, R = stress ratio, and S = maximum stress.

For aluminum fastener hole specimens subjected to spectrum loadings, such as bomber or fighter spectra, extensive fractographic data indicate that the crack growth rate equation can be expressed as (e.g., Refs. 7-10)

$$\frac{\mathrm{d}a(t)}{\mathrm{d}t}=Qa^b(t) \tag{2}$$

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To take into account the statistical variability in the crack growth damage accumulation, Eqs. (1) and (2) have been randomized as follows (e.g., Refs. 2, 11-14):

$$\frac{da(t)}{dt}=X(t)L(\Delta K,a,K,R,S)=X(t)Qa^b(t) \tag{3}$$

in which the additional factor $X(t)$ is a non-negative random process with the median value equal to unity. Thus, the deterministic crack growth rate equation given by Eqs. (1) and (2) represents the median crack growth rate behavior, whereas the statistical variability of the crack growth rate is taken care of by the random process $X(t)$.

It is of interest to note that Virkler et al.[15] have performed simulation studies of crack propagation which amount to assuming $X(t)$ in Eq. (3) to be totally uncorrelated at any two different times, referred to as a white noise process.[14] At the other extreme, Yang et al.[2,12] have investigated the case in which $X(t)$ is totally correlated at all times, i.e., $X(t)=X$ is a random variable, referred to as the random variable model.[14] It was pointed out in Refs. 2 and 14 that a totally uncorrelated $X(t)$, i.e., a white noise process model, would lead to the smallest statistical dispersion, and a totally correlated $X(t)$, i.e., a random variable model, would lead to the greatest statistical dispersion for the time at which a given crack size is reached. In the literature, Lin and Yang[11,13,16] also treated $X(t)$ as a Markov random process, and other stochastic models for fatigue crack propagation have been proposed (e.g., Refs. 17-21).

Mathematically the analysis is simplified significantly by treating $X(t)$ as a random variable X. Further, it may result in a conservative estimation of the crack growth dispersion. This makes the random variable model attractive for practical applications, such as the durability and damage tolerant analyses. Yang and Donath have demonstrated in Ref. 2, using the fractographic data generated in Ref. 1, that the statistical model given by Eq. (3), with $X(t)$ being a random variable X, can be used for fatigue crack propagation in fastener holes under bomber and fighter loading spectra *in the small crack size region* (0.004 to 0.07 in.), i.e.,

$$\frac{da(t)}{dt}=XQa^b(t) \tag{4}$$

Recently General Dynamics/Fort Worth Division fatigue-tested eight dog-bone specimens of 7475-T7351 aluminum with a 3.00-in. width and a 0.375-in. thickness in the test section. These tests were conducted to acquire natural fatigue cracks in fastener holes greater than 0.60 in. Each specimen contained a 0.250-in.-nominal-diam straight-bore center hole with a NAS6204 (0.25-in.-diam) steel protruding head bolt installed with a "finger-tight" nut. All fastener holes were drilled with a modified spacematic drill without deburring holes.

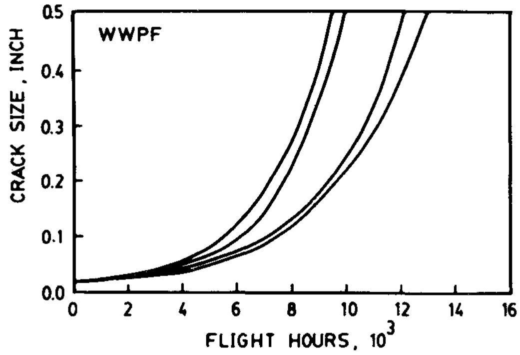

Fig. 1 Actual crack propagation time histories of fastener hole specimens for WWPF data set.

Four specimens were tested under a fighter spectrum, referred to as the WWPF data set, and four other specimens were tested under a bomber spectrum, referred to as the WWPB data set. Fastener holes were not intentionally preflawed in any of the eight specimens tested so that natural fatigue cracks could be obtained. Hence, the time-to-crack-initiation varies from one specimen to another.

The fractographic data [i.e., all $a(t)$ vs t records without a crack size range limitation] for each specimen in the WWPB and WWPF data sets were normalized to a zero life at a crack size of 0.008 in. and 0.017 in., respectively. For the purpose of statistical analysis and correlation study, this normalization procedure is used to obtain homogeneous crack growth data bases in which each specimen starts with the same initial crack size. The normalized crack growth results for the two data sets are presented in Figs. 1 and 2.

Taking the logarithm of both sides of Eq. (4), one obtains

$$Y=bU+q+Z \tag{5}$$

where

$$Y=\log\frac{da(t)}{dt},\quad U=\log a(t),\quad q=\log Q,\quad Z=\log X \tag{6}$$

The relationship of the log crack growth rate Y to the log crack size U, for the test results shown in Figs. 1 and 2, has been obtained using the 5-point incremental polynomial method (e.g., Ref. 22). The results are represented in Figs. 3 and 4 by open circles. It is observed from Figs. 3 and 4 that the test results scatter around a straight line, indicating the validity of Eq. (2).

Crack growth rate data have also been derived from Figs. 1 and 2 using the secant or modified secant method.[14] However,

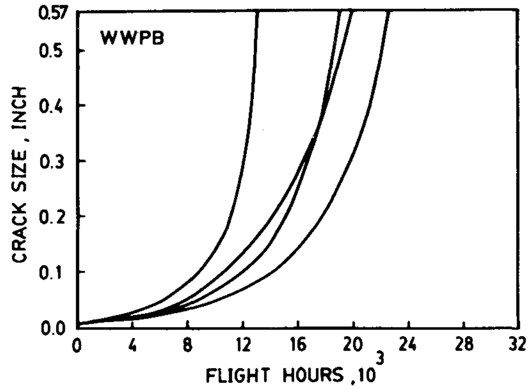

Fig. 2 Actual crack propagation time histories of fastener hole specimens for WWPB data set.

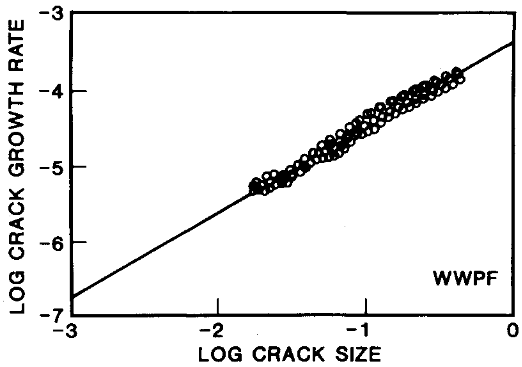

Fig. 3 Crack growth rate as a function of crack size for WWPF data set using 5-point incremental polynomial method.

these two methods are not recommended because they introduce a larger additional statistical dispersion of the crack growth rate than the 5-point incremental polynomial method, as will be discussed later.

In Refs. 11-14, $X(t)$ is modeled as a lognormal random process in the sense that its logarithm is a normal (Gaussian) random process. Hence, in Eq. (4) X is a positive random variable following the lognormal distribution with a median value equal to unity. It follows, then, from Eq. (6) that $Z = \log X$ is a normal random variable with zero mean and standard deviation σ_z. From Eq. (5), the log crack growth rate $Y = \log[\mathrm{d}a(t)/\mathrm{d}t]$ is a normal random variable with the mean value μ_y and standard deviation σ_y given by

$$\mu_y = bU + q \tag{7}$$

$$\sigma_y = \sigma_z \tag{8}$$

The crack growth rate parameters b and Q, as well as the standard deviation σ_z of Z, conditional on the crack size a, can be estimated from the test results of the crack growth rate vs the crack size using Eq. (5) and the linear regression analysis. With the crack growth rate data shown as open circles in Figs. 3 and 4, the method of linear regression is used to estimate b,

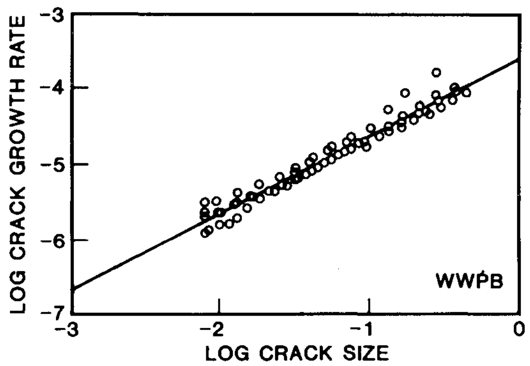

Fig. 4 Crack growth rate as a function of crack size for WWPB data set using 5-point incremental polynomial method.

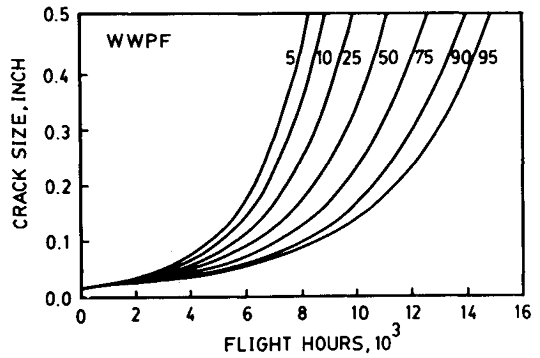

Fig. 5 Percentiles of crack size $a(t)$ as a function of service time t based on statistical model for WWPF data set.

Q, and σ_z. The results are presented in Table 1. The mean of log crack growth rate μ_y given by Eq. (7) is shown in Figs. 3 and 4 as a straight line.

To show that the random variable z follows the normal distribution, i.e., X follows the lognormal distribution, sample values of Z, denoted by z_j ($j = 1,2,\ldots$), were computed from Figs. 3 and 4. When the sample values are plotted on normal probability paper, they scattered around a straight line, indicating an excellent fit for the normal distribution. Further, Kolmogorov-Smirnov tests for goodness-of-fit were conducted, and the results indicated that the normal distribution was acceptable at least at a 20% level of significance. Thus, the validity of the lognormal distribution for X has been demonstrated (see Ref. 14 for detailed results).

The crack growth rate $\mathrm{d}a/\mathrm{d}t$ follows the lognormal distribution and the coefficient of variation, denoted by V, is related to σ_z through the following relation:

$$V = \{\exp[(\sigma_z \ell n 10)^2] - 1\}^{1/2} \tag{9}$$

The coefficients of variation V of the crack growth rate for the WWPF and WWPB data sets are also shown in Table 1.

Equation (4) can be integrated to yield the crack size $a(t)$ as a function of flight hours t,

$$a(t) = a(0)/[1 - XcQta^c(0)]^{1/c} \tag{10}$$

in which $a(0)$ is the initial crack size and

$$c = b - 1 \tag{11}$$

Let z_γ be the γ percentile of the normal random variable Z, i.e.,

$$\gamma\% = P[Z > z_\gamma] = 1 - \Phi(z_\gamma/\sigma_z)$$

or, conversely,

$$z_\gamma = \sigma_z \Phi^{-1}(1 - \gamma\%) \tag{12}$$

in which $\Phi(\;)$ is the standardized normal distribution function and $\Phi^{-1}(\;)$ is the corresponding inverse function.

Then, the γ percentile of the random variable X, denoted by x_γ, is given by

$$x_\gamma = (10)^{z_\gamma} \tag{13}$$

and the γ percentile of the crack size, $a_\gamma(t)$, at t flight hours follows from Eqs. (10) and (13) as

$$a_\gamma(t) = a(0)/[1 - x_\gamma cQta^c(0)]^{1/c} \tag{14}$$

Various γ percentiles of the crack size $a_\gamma(t)$ vs flight hours t have been computed from Eqs. (11-14), using the parameter values given in Table 1 for the WWPF and WWPB data sets. The results are presented in Figs. 5 and 6, in which the initial crack size $a(0)$ for the WWPF data set is 0.017 in. and that for the WWPB data set is 0.008 in. For example, the curve

Table 1 Linear regression estimate of b, Q, σ_z, and coefficient of variation V of crack growth rate; $a(0)$ = initial crack size, a_F = final crack size, CWPF* = method of 5-point incremental polynomial using raw data, CWPF = modified secant method using raw data**

Data set	b	Q, 10^{-3}	σ_z	V, %	$a(0)$, in.	a_F, in.
WWPF	1.123	0.413	0.077	17.9	0.017	0.51
WWPB	1.012	0.237	0.110	25.8	0.008	0.57
CWPF	1.372	2.128	0.202	49.2	0.010	0.35
CWPF*	1.385	2.120	0.219	53.9	0.010	0.35
CWPF**	1.393	2.142	0.231	57.2	0.010	0.35

associated with $\gamma = 10$ in Fig. 5 indicates that the probability is 10% that a WWPF specimen will have a crack growing faster than that shown by the curve.

Thus, on the basis of the statistical model, the distribution function of the crack size, $a(t)$, as a function of service life t (flight hours), is established by Eqs. (12-14) and shown in Figs. 5 and 6 for the WWPF and WWPB data sets, respectively.

For the prediction of crack growth damage accumulation in fastener holes in durability and damage tolerance analyses, two statistical distributions are most important: the distribution of crack size at any service time t and the distribution of service life to reach any given crack size, including the critical crack size. Using the present lognormal random variable model, these two distribution functions can be derived easily in the following manner.

The distribution function of the lognormal random variable X is given by

$$F_X(x) = P[X \leq x] = \Phi(\log x/\sigma_z) \tag{15}$$

in which σ_z has been obtained from the crack growth rate data as shown in Table 1 for the WWPF and WWPB data sets.

The distribution function of the crack size $a(t)$ at any service life, t, can be obtained from that of X given by Eq. (15) through the transformation of Eq. (10). The results are

$$F_{a(t)}(x) = P[a(t) \leq x] = \Phi\left[\log\left(\frac{a^{-c}(0) - x^{-c}}{cQt}\right)\bigg/\sigma_z\right] \tag{16}$$

Let $T(a_1)$ be a random variable denoting the propagation time to reach any given crack size a_1. Then $T(a_1)$ can be obtained from Eq. (10) by setting $a(t) = a_1$ and $t = T(a_1)$, respectively, i.e.,

$$T(a_1) = \frac{1}{cQX}[a^{-c}(0) - a_1^{-c}] \tag{17}$$

Thus the distribution of $T(a_1)$ can be obtained from that of X given by Eq. (15) through the transformation of Eq. (17). The results can be expressed as follows:

$$F_{T(a_1)}(\tau) = P[T(a_1) \leq \tau] = 1 - \Phi[(\log \eta)/\sigma_z] \tag{18}$$

where

$$\eta = (1/cQ\tau)[a^{-c}(0) - a_1^{-c}] \tag{19}$$

In the durability analysis, the extent of cracking in a structural component can be determined from the probability of crack exceedance, i.e., the probability that a crack size may exceed any specific value x_1 at any point in time τ. The probability of crack exceedance, denoted by $p(x_l,\tau)$, is the complement of the distribution function of the crack size $a(\tau)$, i.e.,

$$p(x_1,\tau) = p[a(\tau) > x_1] = 1 - F_{a(\tau)}(x_1) \tag{20}$$

where $F_{a(\tau)}(x_1)$ is given by Eq. (16), with t and x being replaced by τ and x_1, respectively.

It is observed from Eqs. (16-20) that the distribution functions of the crack size at any given number of flight hours, and the time to reach any specific crack size, as well as the probability of crack exceedance derived above, require only the crack growth rate parameters b and Q, as well as the model statistics σ_z, which are determined from the linear regression analysis of the crack growth rate data as presented in Table 1.

III. Correlation Study

Based on the statistical model described above, the distribution function of the crack size, $a(t)$, as a function of flight hours t can be expressed in terms of various γ percentiles. The results for the WWPF and WWPB fastener holes are shown in Figs. 5 and 6, respectively. A comparison between Figs. 1 and 5, as well as between Figs. 2 and 6, indicates a good correlation between the experimental test results and the lognormal random variable model.

Using Eqs. (18) and (19), the distributions for the number of flight hours to reach crack sizes 0.05, 0.15, and 0.51 in. are displayed in Fig. 7 as solid curves for the WWPF data set. Similarly the distributions for the number of flight hours to reach crack sizes 0.025, 0.1, and 0.57 in. for the WWPB fastener holes are shown in Fig. 8. The corresponding experimental test results obtained from Figs. 1 and 2 are plotted in Figs. 7 and 8 as circles. Figures 7 and 8 clearly demonstrate a good correlation between the statistical model (solid curves) and the experimental test results.

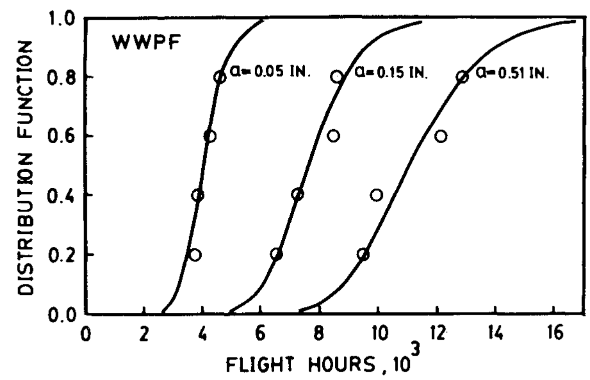

Fig. 7 Correlation between predictions based on lognormal random variable model and test results for the distribution of time to reach crack sizes of 0.05, 0.15, and 0.51 in. for WWPF data set.

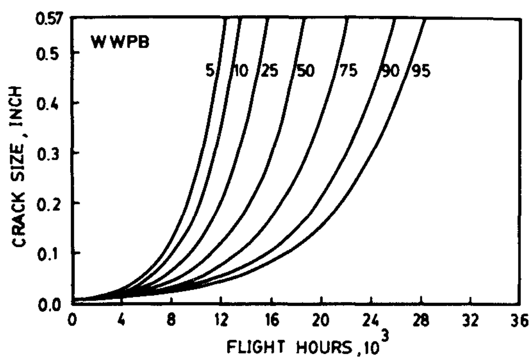

Fig. 6 Percentiles of crack size $a(t)$ as a function of service time t based on statistical model for WWPB data set.

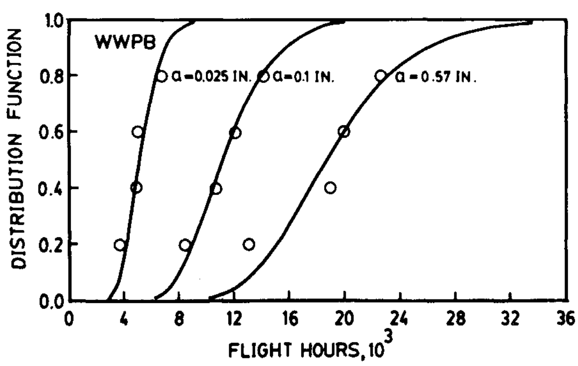

Fig. 8 Correlation between predictions based on lognormal random variable model and test results for the distribution of time to reach crack sizes of 0.025, 0.1, and 0.57 in. for WWPB data set.

The crack exceedance curves based on the theoretical model, Eq. (20), for the WWPF fastener holes at 6000 flight hours and the WWPB fastener holes at 7000 flight hours are plotted in Figs. 9 and 10, respectively, as solid curves. Shown in these figures as circles are the corresponding test results obtained from Figs. 1 and 2. Again, the correlation between the statistical model and the test results is very good.

The correlation study is conducted further using corrosion fatigue fractographic data for dog-bone specimens with a center fastener hole. Recently ten dog-bone specimens (7075 T7651 aluminum) were fatigue-tested in a 3.5% NaCl solution using a fighter spectrum (hi-lo 400 hr block). These tests, sponsored by the Naval Air Development Center (Warminster, PA), were conducted at the General Dynamics/Fort Worth Division. Test and fractographic results are documented in Ref. 3. Test specimens were 2-in. wide and 0.3-in. thick in the test section and included a center hole (open with a nominal diameter of 7/16 in.). All fastener holes were polished to obtain at least an 8-min. finish in the bore of the hole. An environmental chamber containing 3.5% NaCl solution was mounted on the test specimen. All spectrum fatigue tests were run continuously until specimen failure or to a specified time. Servocontrolled hydraulically actuated load frames were used. Two different loading frequencies were used: fast = 8,000 flight hours per 2 days and slow = 8000 flight hours per 16 days. A fractographic evaluation of the largest fatigue crack for each specimen was performed to determine the crack growth behavior in terms of crack size vs flight hours.

Fastener holes were not intentionally preflawed in any of the ten specimens so that natural fatigue cracks could be obtained, and the time-to-crack-initiation varied from one specimen to another. The fractographic data for each specimen were normalized to a zero life at a crack of 0.01 in., and they were referred to as the CWPF data set. For the purpose of statistical analysis and correlation study, this normalization procedure was used to obtain the homogeneous crack growth data base in which each specimen starts with the same initial crack size. The normalized crack growth results are presented in Fig. 11. Observe that the statistical dispersion of the crack growth damage accumulation is very large; a typical phenomenon of corrosion-fatigue cracking in fastener holes.

The log crack growth rate vs the log crack size data have been obtained using the 5-point incremental polynomial method, and the results are presented in Fig. 12 as open circles. A linear regression analysis is used to estimate the crack growth rate parameters b and Q as well as σ_z. The results are given in Table 1. Based on the statistical model, various γ percentiles of the crack size $a_\gamma(t)$ vs flight hours are displayed in Fig. 13. The distribution functions for the number of flight hours to reach crack sizes 0.04, 0.08, and 0.35 in. are shown in Fig. 14 as solid curves. The corresponding experimental test results from Fig. 11 are plotted in Fig. 14 as circles. Furthermore, the crack exceedance curve at 1500 flight hours is depicted as a solid curve in Fig. 15. The experimental test results from Fig. 11 are shown in Fig. 15 as circles. It is observed from Figs. 14 and 15 that the correlation between the stochastic model and the experimental test results is very good.

The simple stochastic model given by Eq. (3), in which $X(t) = X$ is considered a lognormal random variable, has been demonstrated here using the fractographic results of fastener hole specimens. It is mentioned that such a model is also applicable to the crack propagation of gas turbine engine materials in high-temperature environments under cyclic or sustained loads.[12,23-26]

IV. Stochastic Analysis of Test Results

The procedures used in the stochastic crack growth analysis described in the previous sections are summarized in the following: 1) Experimental test results for the crack size $a(t)$ vs cycles t (or flight hours) are measured for many replicate specimens. These test results are referred to as the primary data (see Figs. 1 and 2). 2) The crack growth rate data can be derived from the primary data, using various data processing procedures (see Figs. 3 and 4). 3) All the derived crack growth rate data are pooled and the linear regression analysis is performed to estimate the crack growth rate parameters b and Q, as well as σ_z, which is a measure of the crack growth rate dispersion (see Table 1). In the second step above, various data processing procedures may be used.[22] However, each procedure results in different crack growth rate data. In the third step, bias in determining the crack growth rate parameters may be induced by the number of data points associated with each test specimen. These problems will be addressed in this section.

Equal Number of Data Points for Each Specimen

Since the fatigue crack propagation involves considerable statistical variability, some specimens may have short prop-

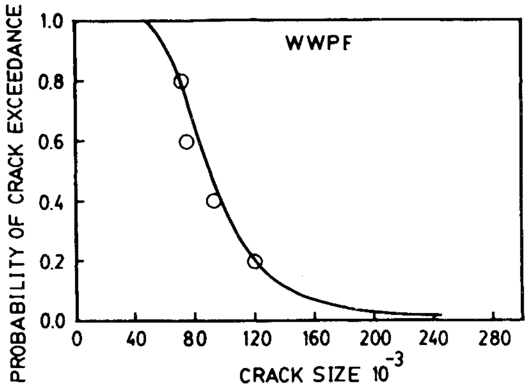

Fig. 9 Correlation between predictions based on lognormal random variable model and test results for the probability of crack exceedance at 6000 flight hours for WWPF data set.

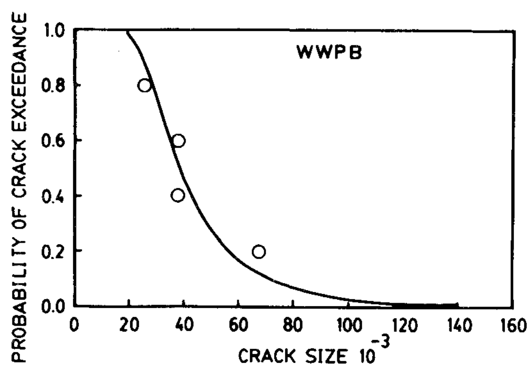

Fig. 10 Correlation between predictions based on lognormal random variable model and test results for the probability of crack exceedance at 7000 flight hours for WWPB data set.

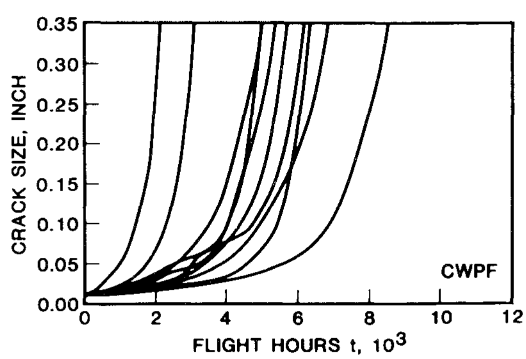

Fig. 11 Actual crack propagation time histories of fastener hole specimens for CWPF data set.

agation lives while others have longer lives. Therefore, more crack size measurements [i.e., $a(t)$ vs t readings] may be taken for slow crack growth specimens than for fast crack growth ones. This is particularly true for the fractographic readings of fastener holes, where fatigue tests are conducted on specimens without an intentional preflaw. In fact, all the fractographic data sets investigated previously do not have an equal number of data points for each specimen.

When such primary data are processed and the resulting crack growth rate data are pooled together for the linear regression analysis, the estimated crack propagation parameters, such as b and Q, are biased to the slow crack growth rate. This is because more data points are usually measured for the slow crack growth specimens. As such, it clearly violates the statistical premise that each specimen (a sample) is of equal weight. Consequently, the resulting statistical fatigue crack propagation predictions are biased toward the unconservative nature, i.e., the stochastic model tends to predict longer fatigue life or smaller crack size.

To circumvent such an error due to an unequal number of measurements for each specimen, additional data points for the primary data, i.e., $a(t)$ vs t, are suggested to be added artificially to the fast crack growth specimens. The idea is to equalize the number of data points for each specimen. In most cases, the artificial points can be determined by interpolation. However, in some cases additional data points may be needed outside the region of available primary data, and extrapolation procedures may not be satisfactory. In this case it is suggested that the primary data for a particular specimen be best fitted using the crack propagation model, i.e., Eq. (2). Then, the additional data points outside the available primary data region are determined from the model.

To demonstrate such a crucial point, consider the CWPF data set. Crack growth rate data derived directly from the fractographic readings using the 5-point incremental polynomial method are used to estimate the crack propagation parameters b and Q, as well as the standard deviation of the log crack growth rate σ_z. The results are presented in Table 1 as CWPF*. In Table 1 the results for WWPF, WWPB, and CWPF data sets reflect additional data points which have been added artificially to those specimens with fast crack growth rates to equalize the number of data points in the applicable crack size range. Based on the stochastic model, the distribution functions for the random time to reach some specific crack sizes are shown in Fig. 14 as dashed curves. The solid curves in the figure are the corresponding results with added data points, and the circles are the experimental test results. As expected, the dashed curves are biased toward the slow crack growth, and hence their correlations with the experimental test results are not as good as the solid curves.

Data Processing Procedures

Various data processing procedures, including the secant method, modified secant method, and 3-, 5-, 7-, and 9-point incremental polynomial methods, have been proposed in the literature[22] to obtain the crack growth rate data from the primary data. Unfortunately each data processing procedure

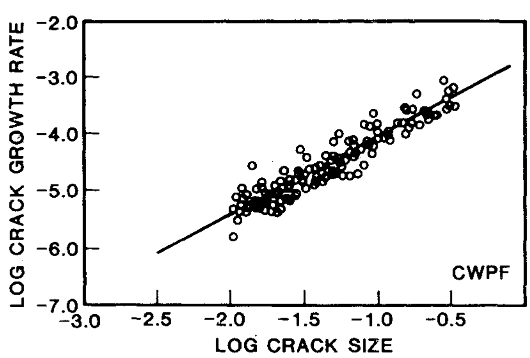

Fig. 12 Crack growth rate as a function of crack size for CWPF data set using 5-point incremental polynomial method.

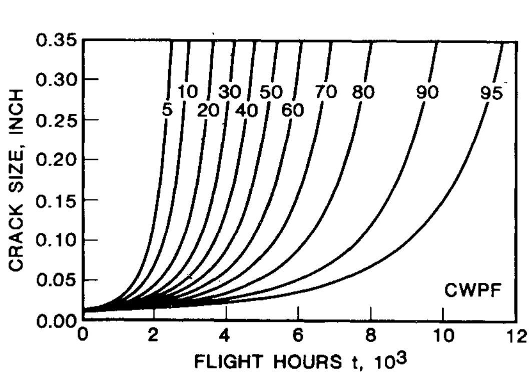

Fig. 13 Percentile of crack size $a(t)$ as a function of service time t based on statistical model for CWPF data set.

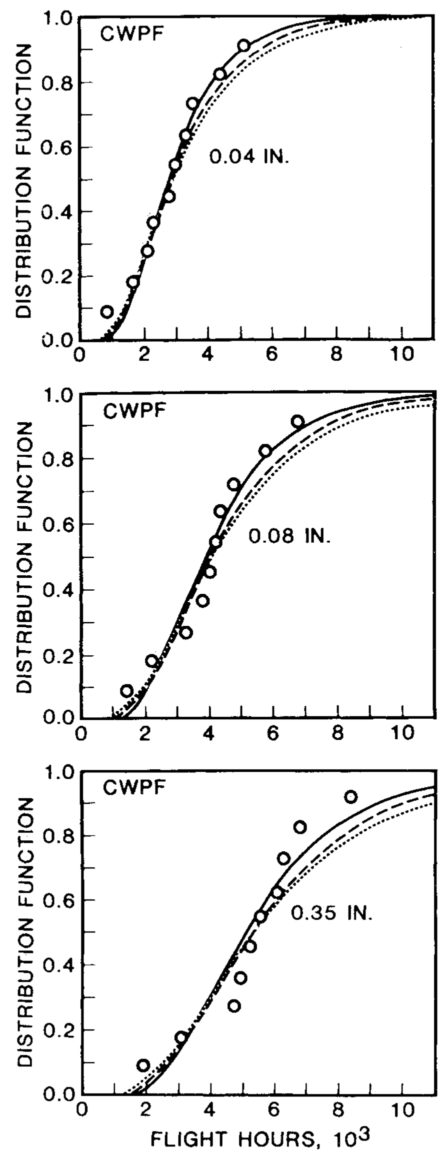

Fig. 14 Correlation between predictions based on lognormal random variable model and test results for the distribution of time to reach crack sizes 0.04, 0.08, and 0.35 in. for CWPF data set: ___ = 5-point incremental polynomial method with added data; ---- = 5-point incremental polynomial method with raw data; .... = modified secant method.

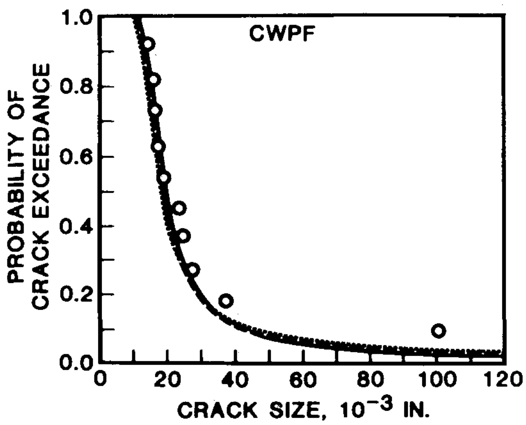

Fig. 15 Correlation between prediction based on lognormal random variable model and test results for the probability of crack exceedance at 1500 flight hours for CWPF data set.

introduces additional statistical variability into the crack growth rate data. Results of the investigation of this subject will appear elsewhere. It is found that the secant method introduces a much larger additional statistical dispersion for the crack growth rate data than any of the incremental polynomial methods. This has been expected because the method of incremental polynomial tends to smooth out the data. The induced undesirable statistical variability for the crack growth rate data reduces slightly as more points are used in the incremental polynomial such as nine or seven points. While it may be desirable to use the 7- or 9-point incremental polynomial method, limited amounts of data available may inhibit its application. As a result, the 5-point incremental polynomial method appears to be quite reasonable.

Again, the CWPF data set is considered for illustrative purposes. With the application of the modified secant method, the estimated parameters b and Q, the standard deviation σ_z, and the coefficient of variation V of the crack growth rate are shown in Table 1 as CWPF** for comparison. Based on the stochastic model, the distribution functions of the random time to reach some specific crack sizes are displayed in Fig. 14 as dotted curves. It is observed from Fig. 14 that the modified secant method introduces a large additional statistical dispersion and hence its correlation with the experimental test results (circles) is not as good as the 5-point incremental polynomial method (solid curves). Similar behaviors have been observed in all other data sets. Finally, poorer correlations are obtained using the secant method than the modified secant method. It is concluded that, for the stochastic crack growth analysis, the method of 5-point incremental polynomial is superior to both the secant and modified secant methods.

V. Conclusions

It is demonstrated in this paper that the simple lognormal random variable model for the fatigue crack growth rate is very reasonable for describing the fatigue crack growth damage accumulation in fastener holes under either fighter or bomber loading spectra. Also, this simple stochastic crack growth model worked very well for both laboratory air and 3.5% NaCl environments. The stochastic model is valid in both the small and large crack size regions.

For the stochastic crack growth analysis, it is demonstrated that the methods of incremental polynomials are superior to the secant methods, since the latter introduce more undesirable statistical variability to the crack growth rate data. Furthermore, the importance of having an equal number of data points for each specimen in the stochastic crack propagation analysis has been illustrated. A method has been suggested and evaluated for equalizing the number of data points for each specimen.

The linear regression estimate for a few crack growth rate parameters and the model statistic requires only the crack growth rate data. How the crack size varies as a function of propagation life is not needed. Hence, the model requires neither a large number of test specimens nor a homogeneous data set for an effective assessment of the crack growth damage accumulation. As a result, the present model is suitable for practical applications in view of the limited experimental data normally available. In addition, because of its mathematical simplicity and conservative nature, such a stochastic model is appropriate for applications to durability and damage tolerant analyses.[14]

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